

通信1402212  
张译同

4.6. 求下列周期信号的基波角频率 $\Omega$ 和周期 $T$ .

(1)  $e^{j100t}$

解:  $= \frac{1}{2}(\cos 100t + j\sin 100t)$

$\therefore \Omega = 100 \text{ rad/s}, T = \frac{2\pi}{\Omega} = \frac{\pi}{50} \text{ s}$

(2)  $\cos(2t) + \sin(4t)$

解:  $\Omega = 2 \text{ rad/s}$

$T = \frac{2\pi}{\Omega} = \pi \text{ s}$

$T = \frac{2\pi}{\Omega}$

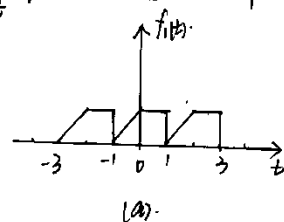
(5)  $\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{4}t)$

解:  $\Omega = \frac{\pi}{4} \text{ rad/s}, T = \frac{2\pi}{\Omega} = 8 \text{ s}$

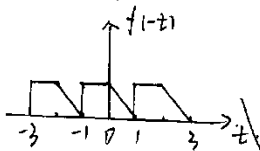
$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos n\omega t dt$

$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin n\omega t dt$

$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega t} dt$



解: 由图知  $f(-t)$  波形为:

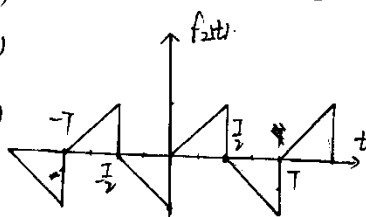


任意函数都可以表示为奇函数和偶函数的两部分。

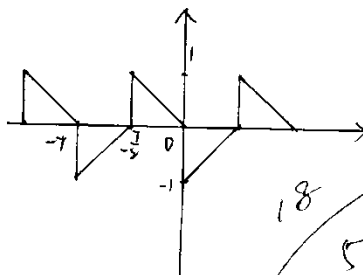
$f(t) = f_{od}(t) + f_{ed}(t)$

奇:  $f_{od}(t) = \frac{f(t) - f(-t)}{2}$

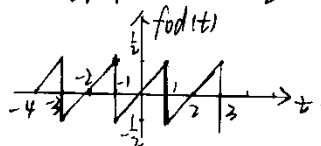
偶:  $f_{ed}(t) = \frac{f(t) + f(-t)}{2}$



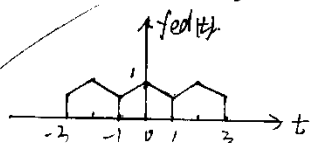
解: 可得  $f_2(-t)$  波形:



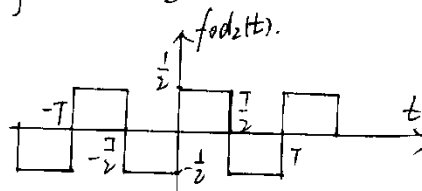
$\therefore$  奇分量波形为  $f_{od}(t) = \frac{f_1(t) - f_1(-t)}{2}$



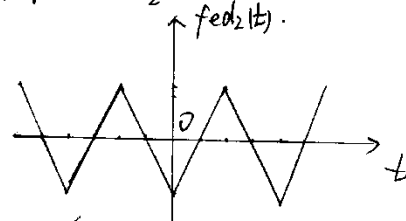
偶分量波形  $f_{ed}(t) = \frac{f_1(t) + f_1(-t)}{2}$



奇:  $f_{od}(t) = \frac{f_2(t) - f_2(-t)}{2}$



偶:  $f_{ed} = \frac{f_2(t) + f_2(-t)}{2}$



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5-2016

# 第四章 傅里叶变换和系统频域分析

## 习题四

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201414107

知识回归纳:

① 三角函数集

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

( $a_n$  是偶函数,  $b_n$  是奇函数)

$$f(t) = \frac{A_0}{2} + A_1 \cos(\omega t + \varphi_1) + A_2 \cos(2\omega t + \varphi_2) + \dots + A_n \cos(n\omega t + \varphi_n)$$

$A_0$ : 直流分量

$A_1 \cos(\omega t + \varphi_1)$ : 基波

$A_2 \cos(2\omega t + \varphi_2)$ : 2次谐波

$A_n \cos(n\omega t + \varphi_n)$ :  $n$ 次谐波

② 奇偶函数傅里叶级数

偶函数:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t)$$

奇函数:

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

奇谐函数:

$$f(t) = \sum_{n=1,3,5,\dots}^{\infty} a_n \cos(n\omega t) + \sum_{n=1,3,5,\dots}^{\infty} b_n \sin(n\omega t)$$

偶谐函数:

$$f(t) = \frac{a_0}{2} + \sum_{n=2,4,6,\dots}^{\infty} a_n \cos(n\omega t) + \sum_{n=2,4,6,\dots}^{\infty} b_n \sin(n\omega t)$$

正弦余弦 余弦正弦

4.6 求下列周期信号基波角频率  $\Omega$  和周期  $T$ .

(1)  $e^{j100t}$

半: 角频率  $\Omega = 100 \text{ rad/s}$ , 周期  $T = \frac{2\pi}{\Omega} = \frac{2\pi}{100} = \frac{\pi}{50} \text{ s}$

(2)  $\cos(2t) + \sin(4t)$

半:  $\cos(2t)$  的角频率  $\Omega_1 = 2 \text{ rad/s}$ ,  $\sin(4t)$  的角频率  $\Omega_2 = 4 \text{ rad/s}$ .

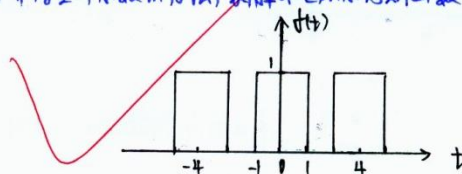
则  $\Omega = 2 \text{ rad/s}$ , 周期  $T = \frac{2\pi}{\Omega} = \pi \text{ s}$ .

(3)  $\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{4}t)$

半:  $\cos(\frac{\pi}{2}t)$  的角频率  $\Omega_1 = \frac{\pi}{2} \text{ rad/s}$ ,  $\sin(\frac{\pi}{4}t)$  的角频率  $\Omega_2 = \frac{\pi}{4} \text{ rad/s}$ .

则  $\Omega = \frac{\pi}{4} \text{ rad/s}$ , 周期  $T = 2\pi \cdot \frac{4}{\pi} = 8 \text{ s}$ .

4.7 用直接计算傅里叶系数方法, 求解如图示周期函数傅里叶系数



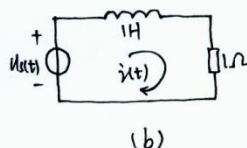
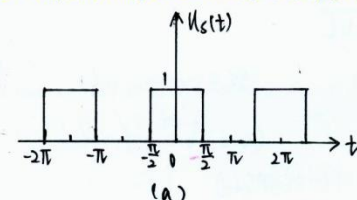
半: 由图可知  $T=4$ , 角频率  $\Omega = \frac{2\pi}{T} = \frac{\pi}{2}$

则指数傅里叶系数为  $F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\Omega t} dt = \frac{1}{4} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\frac{\pi}{2}t} dt$

$$= \frac{1}{4} \int_{-1}^1 e^{-jn\frac{\pi}{2}t} dt = \frac{1}{4} \cdot \frac{2}{-jn\pi} e^{-jn\frac{\pi}{2}t} \Big|_{-1}^1$$

$$= \frac{1}{-j2n\pi} (e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}}) = \frac{\sin(\frac{n\pi}{2})}{n\pi}, (n=0, \pm 1, \pm 2, \dots)$$

4.12 如图示用周期性方波电压作用于RL电路, 试求电流  $i(t)$  的前五次谐波.



半: 由图 (a) 知  $u_s(t)$  的周期  $T=2\pi \text{ s}$ , 角频率  $\Omega = \frac{2\pi}{T} = 1 \text{ rad/s}$ .

$u_s(t)$  在  $(-\frac{T}{2}, \frac{T}{2})$  区间内表达式可写为  $u_s(t) = \begin{cases} 1, & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0, & -\pi < t < -\frac{\pi}{2}, \frac{\pi}{2} < t < \pi \end{cases}$

$\because u_s(t)$  是偶函数, 由傅里叶系数公式

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_s(t) dt = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u_s(t) dt = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 dt = 1$$

$$a_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_s(t) \cos(n\Omega t) dt = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(n\Omega t) dt = \frac{1}{n\pi} \sin(\frac{n\pi}{2}), n=1, 2, 3, \dots$$

$$b_n = 0, n=1, 2, 3, \dots$$

5-2016

# 第四章 傅里叶变换和系统的频域分析

通信工程1401班

顾曼璇

2014/7/15

基波角频率

$$\Omega = \frac{2\pi}{T}$$

4.6.  $10e^{j100t}$

1) 角频率  $\Omega = 100 \text{ rad/s}$ . 周期  $T = \frac{2\pi}{\Omega} = \frac{2\pi}{100} = \frac{\pi}{50} \text{ s}$ .

2)  $\cos(2t) + \sin(4t)$ .

$\cos 2t$  角频率  $\Omega_1 = 2 \text{ rad/s}$ .  $\sin 4t$  角频率  $\Omega_2 = 4 \text{ rad/s}$ .

最大角频率.  $\cos 2t + \sin 4t$  基波角频率  $\Omega = 2 \text{ rad/s}$ .

$$T = \frac{2\pi}{\Omega} = \frac{2\pi}{2} = \pi \text{ s}$$

3)  $\cos(\frac{\pi}{2}t) + \sin(\frac{1}{4}t)$ .

$\cos(\frac{\pi}{2}t)$  角频率  $\Omega_1 = \frac{\pi}{2}$ .  $\sin(\frac{1}{4}t)$  角频率  $\Omega_2 = \frac{1}{4}$ .

最大角频率为  $\frac{\pi}{4}$ .

信号  $\cos(\frac{\pi}{2}t) + \sin(\frac{1}{4}t)$  基波角频率  $\Omega = \frac{\pi}{4} \text{ rad/s}$ .

$$T = \frac{2\pi}{\Omega} = 8 \text{ s}$$

傅里叶级数

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\Omega t}$$

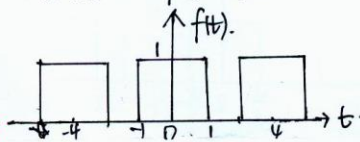
系数  $F_n$  和傅里叶级数

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\Omega t} dt$$

欧拉公式

$$\begin{cases} \cos x = \frac{e^{jx} + e^{-jx}}{2} \\ \sin x = \frac{e^{jx} - e^{-jx}}{2j} \end{cases}$$

4.7. 用直接计算傅里叶级数的方法, 求题图所示周期函数的傅里叶级数。(用欧拉公式表示)



解: 由  $f(t)$  波形, 知周期  $T=4$ . 角频率  $\Omega = \frac{2\pi}{T} = \frac{\pi}{2}$ .

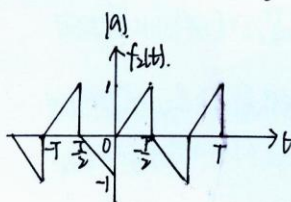
傅里叶级数展开式

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\Omega t} dt = \frac{1}{4} \int_{-2}^2 f(t) e^{-jn\frac{\pi}{2}t} dt$$

$$= \frac{1}{4} \int_{-2}^2 e^{-jn\frac{\pi}{2}t} dt = \frac{1}{4} \cdot \frac{2}{-jn\pi} e^{-jn\frac{\pi}{2}t} \Big|_{-2}^2$$

$$= \frac{1}{-jn\pi} (e^{-jn\pi} - e^{jn\pi}) = n \cdot \frac{1}{\pi} \sin(\frac{n\pi}{2}) \quad n=0, \pm 1, \pm 2, \dots$$

4.9. 试画出题4.9图所示信号的奇分量和偶分量.



$f_{od}(t)$   $f_{ev}(t)$  合称为  $f(t)$  的奇分量和偶分量.

$$f_{od}(t) = \frac{1}{2} [f(t) - f(-t)]$$

$$f_{ev}(t) = \frac{1}{2} [f(t) + f(-t)]$$

(a) 对  $f(t)$  的波形反折运算  $\rightarrow f(-t)$ .

根据上面两式得出.

$$f_{od}(t), f_{ev}(t), f(t) \text{ 波形如下.}$$

(b) 对  $f(t)$  反折得  $f(-t)$

$$f_{2od}(t), f_{2ev}(t) \text{ 波形如下}$$

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5-2016



# 1. 傅里叶级数

## ① 三角形式

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\omega t) dt$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\omega t) dt$$

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \varphi_n)$$

$$\text{式中 } A_0 = a_0 \quad A_n = \sqrt{a_n^2 + b_n^2}$$

$$\varphi_n = -\arctan \frac{b_n}{a_n} \quad n=1, 2, \dots$$

## ② 指数形式

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega t}$$

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega t} dt$$

$F_n$  称为傅里叶系数

$$F_n = |F_n| e^{j\varphi_n} = \frac{1}{2} A_n e^{j\varphi_n} = \frac{1}{2} (a_n - jb_n)$$

## 2. 奇偶分量

对实信号而言

$$f(t) = f_e(t) + f_o(t) \rightarrow \begin{cases} f_e(t) \text{ 偶分量} \\ f_o(t) \text{ 奇分量} \end{cases}$$

$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

$$f_e(t) = \frac{1}{2} [f(t) + f(-t)]$$

$$f_o(t) = \frac{1}{2} [f(t) - f(-t)]$$

## 3. 周期信号 $f(t)$

周期为  $T$

$$\text{角频率为 } \omega = 2\pi f = \frac{2\pi}{T}$$

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5-2016

# 第四章 傅里叶变换和系统的频域分析

4.6. 求下列周期信号的基波角频率  $\omega$  和周期  $T$ .

(1)  $e^{j100t}$

解:  $\omega = 100 \text{ rad/s}$   
 $T = \frac{2\pi}{\omega} = \frac{2\pi}{100} = \frac{\pi}{50} \text{ s}$

(2)  $\cos(2t) + \sin(4t)$

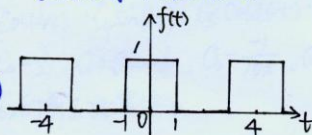
解:  $T_1 = \frac{2\pi}{2} = \pi \quad T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$   
 $\frac{T_1}{T_2} = 2$  有理,  $\therefore T = \frac{\pi}{2} = \pi \text{ s}$   
 $\omega = \frac{2\pi}{T} = 2 \text{ rad/s}$

(5)  $\cos(\frac{\pi}{4}t) + \sin(\frac{\pi}{8}t)$

解:  $T_1 = \frac{2\pi}{\frac{\pi}{4}} = 8 \quad T_2 = \frac{2\pi}{\frac{\pi}{8}} = 16$   
 $\frac{T_1}{T_2} = \frac{1}{2}$  有理,  $T = \frac{8 \times 16}{4} = 32 \text{ s}$   
 $\omega = \frac{2\pi}{T} = \frac{\pi}{16} \text{ rad/s}$

4.7. 用直接计算傅里叶系数的方法, 求下图所示周期函数的傅里叶系数.

(三角形式或指数形式)



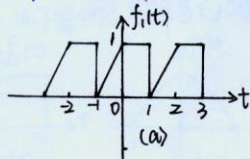
解: 由波形可知,  $T=4 \quad \omega = \frac{2\pi}{T} = \frac{\pi}{2}$   
根据指数傅里叶系数定义式, 可得

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega t} dt$$

$$= \frac{1}{4} \int_{-2}^2 e^{-jn\frac{\pi}{2}t} dt = \frac{1}{4} \left( \frac{-2}{jn\frac{\pi}{2}} \right) e^{-jn\frac{\pi}{2}t} \Big|_{-2}^2$$

$$= -\frac{1}{jn\pi} (e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}) = \frac{1}{jn\pi} \sin(n\frac{\pi}{2}), n=0, \pm 1, \dots$$

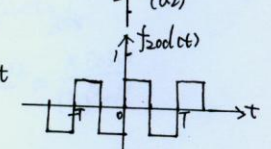
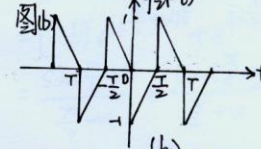
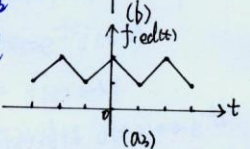
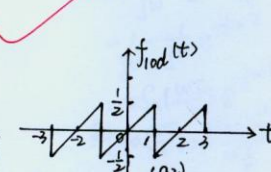
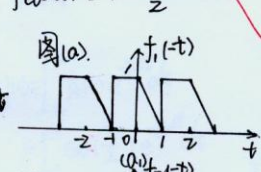
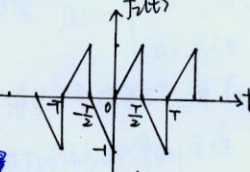
4.9. 试画出下图所示信号的奇偶分量



解:  $f_{od}(t)$  表示奇函数部分  
 $f_{ed}(t)$  表示偶函数部分

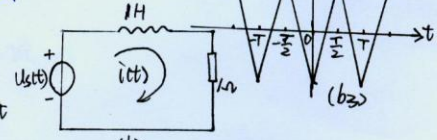
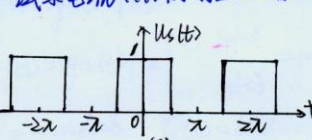
$$f_{od}(t) = \frac{f(t) - f(-t)}{2}$$

$$f_{ed}(t) = \frac{f(t) + f(-t)}{2}$$



4.12. 如图所示的周期性三角波电压作用于RL电路.

试求电流  $i(t)$  的前五次谐波.



解: 由图(a)写出  $u_s(t)$  在  $(-\frac{T}{2}, \frac{T}{2})$  区间的表达式为

$$u_s(t) = \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & -\pi < t < -\frac{T}{2}, \frac{T}{2} < t < \pi \end{cases}$$

$$T=2\pi \text{ s} \quad \omega = \frac{2\pi}{T} = 1 \text{ rad/s}$$

数定义

可知  $u_s(t)$  是偶函数, 根据三角形式傅里叶系

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_s(t) dt = \frac{1}{2} \int_{-1}^1 u_s(t) dt = \frac{1}{2} \int_{-1}^1 1 dt = 1$$

$$a_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u_s(t) \cos(n\omega t) dt = \frac{1}{2} \int_{-1}^1 \cos(n\pi t) dt = \frac{2}{n\pi} \sin(n\frac{\pi}{2}), n=1, 2, 3, \dots$$

月题4.

11).  $e^{j100t} = \cos(100t) + j\sin(100t)$

$T = \frac{2\pi}{100} = \frac{\pi}{50} \text{ s}$      $\Omega = 100 \text{ rad/s}$

13)  $\cos(2t) : T_1 = \frac{2\pi}{2} = \pi \text{ s}$

$\cos(4t) : T_2 = \frac{2\pi}{4} = \frac{\pi}{2} \text{ s}$

$T = \pi \text{ s}$

$\Omega = \frac{2\pi}{\pi} = 2 \text{ rad/s}$

15)  $\cos(\frac{\pi}{5}t) : T_1 = 4 \text{ s}$

$\sin(\frac{\pi}{4}t) : T_2 = 8 \text{ s}$

$T = 8 \text{ s}$

$\Omega = \frac{2\pi}{8} = \frac{\pi}{4} \text{ rad/s}$

4.7.

(a) 由图和  $T = 4$      $\Omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$F_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\Omega t} dt = \frac{1}{4} \int_{-2}^2 f(t) e^{-jn\frac{\pi}{2}t} dt$

$= \frac{1}{4} \int_{-2}^2 e^{-jn\frac{\pi}{2}t} dt = j \frac{1}{2n\pi} (e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}})$

$= \frac{\sin(n\frac{\pi}{2})}{n\pi} \quad n = 0, \pm 1, \pm 2, \dots$

欧拉公式

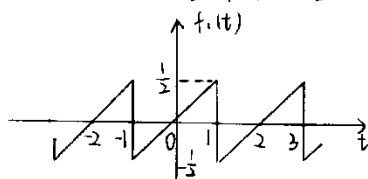
$$\begin{cases} e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \\ e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t) \end{cases}$$

电子1401

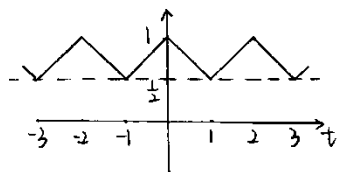
王颖

201414015

4.9 (a)  $f_1(t)$  的奇分量和偶分量分别如图 (a) (b)



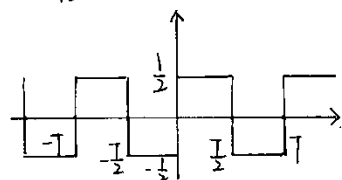
(a)



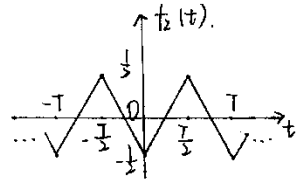
(b)

$f_1(t)$  为偶函数  
展开式只含余弦分量

(b)  $f_2(t)$  的奇分量和偶分量分别如图 (c) (d)



(c)



(d)

$f_2(t)$  为奇函数  
展开式只含正弦分量

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6-2015



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电子信息工程 1403

知识点:

1. 傅里叶级数的三角形式:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

角频率为  $\omega$ , 周期为  $T = \frac{2\pi}{\omega}$

$a_n, b_n$  称为傅里叶系数

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\omega t) dt$$

$n = 0, 1, 2, \dots$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\omega t) dt$$

$n = 0, 1, 2, \dots$

2. 傅里叶级数的指数形式:

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega t}$$

$F_n$  为傅里叶系数

$$F_n = \frac{1}{T} A e^{jn\omega t}$$

$$= \frac{1}{2} (A \cos \varphi_n + j A \sin \varphi_n)$$

$$= \frac{1}{2} (a_n - j b_n)$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega t} dt$$

$n = (0, \pm 1, \pm 2, \dots)$

第4章. 傅里叶变换和系统的频域分析.

4.6. 求下列周期信号的基波角频率  $\omega$  和周期  $T$ .

1)  $e^{j100t}$  (3)  $\cos(2t) + \sin(4t)$  (5)  $\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{4}t)$

解 1): 角频率  $\omega = 100 \text{ rad/s}$ , 周期  $T = \frac{2\pi}{\omega} = \frac{2\pi}{100} = \frac{\pi}{50} \text{ s}$ .

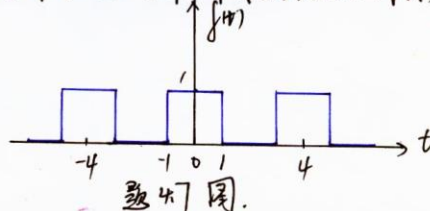
(3):  $\cos(2t)$  的角频率为  $\omega_1 = 2 \text{ rad/s}$ ,  $\sin(4t)$  的角频率为  $\omega_2 = 4 \text{ rad/s}$ , 两者的最大公约数为  $\omega = 2 \text{ rad/s}$ .

则周期  $T = \frac{2\pi}{\omega} = \pi \text{ s}$ .

(5):  $\cos(\frac{\pi}{2}t)$  的角频率为  $\omega_1 = \frac{\pi}{2} \text{ rad/s}$ ,  $\sin(\frac{\pi}{4}t)$  的角频率为  $\omega_2 = \frac{\pi}{4} \text{ rad/s}$ , 两者的最大公约数为  $\omega = \frac{\pi}{4} \text{ rad/s}$ .

所以周期  $T = \frac{2\pi}{\omega} = 8 \text{ s}$ .

4.7. 用直接计算傅里叶系数的方法, 求题4.7图所示周期函数的傅里叶系数 (三角形式或指数形式)



解: 由题图可知该函数的周期  $T = 4$ , 则角频率为  $\frac{2\pi}{T} = \frac{\pi}{2}$ .

利用傅里叶系数定义式, 得指数傅里叶系数

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega t} dt = \frac{1}{4} \int_{-4}^2 f(t) e^{-jn\frac{\pi}{2}t} dt$$

$$= \frac{1}{4} \int_{-4}^0 1 \cdot e^{-jn\frac{\pi}{2}t} dt$$

$$= \frac{1}{4} \cdot \frac{2}{-jn\pi} e^{-jn\frac{\pi}{2}t} \Big|_{-4}^0$$

$$= \frac{1}{j2n\pi} (e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}}) = \frac{\sin(\frac{n\pi}{2})}{n\pi} \quad n = 0, \pm 1, \pm 2, \dots$$

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杨雨婷

$$T = \frac{2\pi}{\Omega}$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\Omega t) dt$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\Omega t) dt$$

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\Omega t} dt$$

$$f(t) = f_{od}(t) + f_{ev}(t)$$

奇函数部分

$$f_{od}(t) = \frac{f(t) - f(-t)}{2}$$

偶函数部分

$$f_{ev}(t) = \frac{f(t) + f(-t)}{2}$$

4.6 求下列周期信号的基本角频率 $\Omega$ 和周期 $T$

1)  $e^{j100t}$

解: 角频率为 $\Omega = 100 \text{ rad/s}$ , 周期 $T = \frac{2\pi}{\Omega} = \frac{\pi}{100} \text{ s}$

3)  $\cos(2t) + \sin(4t)$

角频率为 $\Omega = \frac{\pi}{2} \text{ rad/s}$ , 周期 $T = \frac{2\pi}{\Omega} = 4 \text{ s}$

5)  $\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{4}t)$

角频率为 $\Omega = \frac{\pi}{4} \text{ rad/s}$ , 周期 $T = \frac{2\pi}{\Omega} = 8 \text{ s}$

4.7 用直接计算傅里叶系数的方法, 求周期函数 $f(t)$ 的傅里叶系数(三角函数或指数函数)

解: 周期 $T = 4$ ,  $\Omega = \frac{2\pi}{T} = \frac{\pi}{2}$

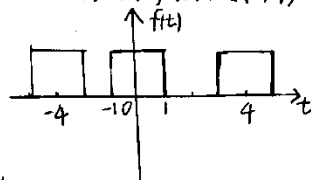
$$f(t) = \begin{cases} 1, & 4k-1 \leq t \leq 4k+1 \\ 0, & 4k+1 < t < 4k+3 \end{cases}$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\Omega t) dt = \frac{1}{2} \int_{-2}^2 f(t) \cos(\frac{n\pi t}{2}) dt$$

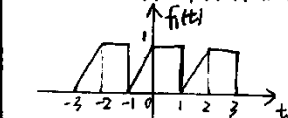
$$= \frac{1}{2} \int_{-1}^1 \cos(\frac{n\pi t}{2}) dt = \frac{2}{n\pi} \sin(\frac{n\pi}{2}), n=0, 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\Omega t) dt = \frac{1}{2} \int_{-2}^2 f(t) \sin(\frac{n\pi t}{2}) dt$$

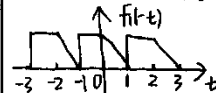
$$= \frac{1}{2} \int_{-1}^1 \sin(\frac{n\pi t}{2}) dt = 0, n=1, 2, \dots$$



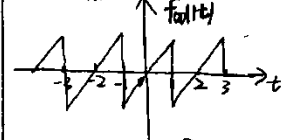
4.9 试画出题4.9图所示信号的奇分量和偶分量



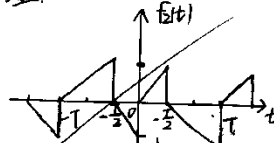
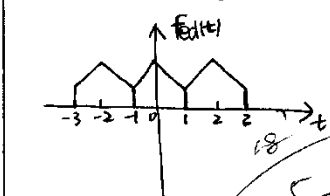
解: 由 $f(t)$ 的波形可得 $f(-t)$ 的波形为



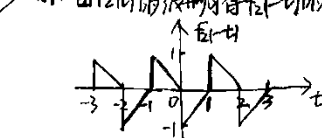
∴ 奇分量 $f_{od}(t) = \frac{f(t) - f(-t)}{2}$ 的波形为



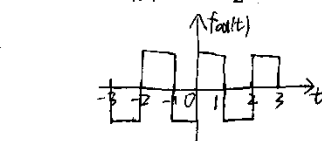
偶分量 $f_{ev}(t) = \frac{f(t) + f(-t)}{2}$ 的波形为



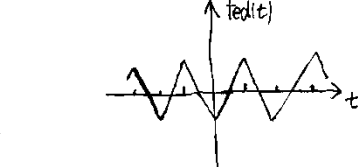
解: 由 $f(t)$ 的波形可得 $f(-t)$ 的波形为



∴ 奇分量 $f_{od}(t) = \frac{f(t) - f(-t)}{2}$ 的波形为



偶分量 $f_{ev}(t) = \frac{f(t) + f(-t)}{2}$ 的波形为



5-2016



通信1401班

史文惠

知识点总结

1.  $T = \frac{2\pi}{\omega}$

周期信号的 $\omega$ 为  
信号中各频率成分  
中频率最小的信号的  
频率,且其余信号  
的角频率均为此  
角频率的整数倍

二、任意函数都可  
以表示为奇函数  
和偶函数两部分

即  $f(t) = f_{od}(t) + f_{ev}(t)$

奇:  $f_{od}(t) = \frac{f(t) - f(-t)}{2}$

偶:  $f_{ev}(t) = \frac{f(t) + f(-t)}{2}$

## 第四章 傅里叶变换和系统的频域分析

Pro1-4.6, 求下列周期信号的基本角频率 $\omega$ 和周期 $T$ .

11).  $e^{j100t}$

解:  $= \frac{1}{2}(\cos 100t + j\sin 100t)$

$\therefore \omega = 100 \text{ rad/s}$

$T = \frac{2\pi}{\omega} = \frac{\pi}{50} \text{ s}$

12).  $\cos(2t) + \sin(4t)$

解:  $\omega = 2 \text{ rad/s}$

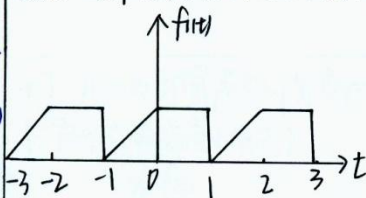
$T = \frac{2\pi}{\omega} = \pi \text{ s}$

15).  $\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{4}t)$

解:  $\omega = \frac{\pi}{4} \text{ rad/s}$

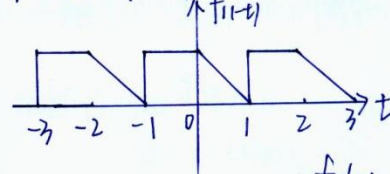
$T = \frac{2\pi}{\omega} = 8 \text{ s}$

Pro2-4.9. 试画出题4.9图所示信号的奇分量和偶分量.

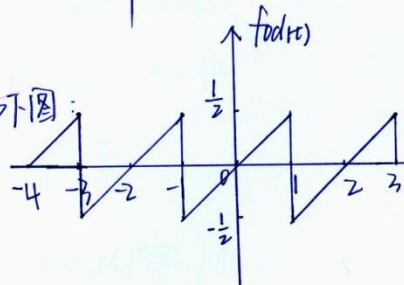


1a).

解: 由图a可知  $f(-t)$  波形为:



$\therefore$  奇分量波形  $f_{od}(t) = \frac{f(t) - f(-t)}{2}$ , 如下图:



偶分量波形  $f_{ev}(t) = \frac{f(t) + f(-t)}{2}$ , 如下图:

